



An Introduction to Algebraic Topology and Topological Data Analysis

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What is a topological space?

What is a Homotopy ?

Definition 2.8

Given two continuous maps $f, f' : X \rightarrow Y$ between topological spaces X and Y , we say that f is **homotopic** to f' , written $f \simeq f'$, if there exists a continuous map $F : X \times [0, 1] \rightarrow Y$ such that $F(x, 0) = f(x)$ and $F(x, 1) = f'(x)$. The map F is called a **homotopy** between f and f' . If f' is a constant map and $f \simeq f'$, then f is said to be **nullhomotopic**.

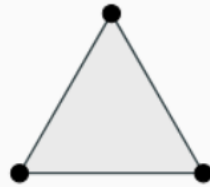
Simplicial complexes

A **k-simplex** is the convex hull of $k + 1$ affinely independent in $\mathbb{R}^{k+1}$.

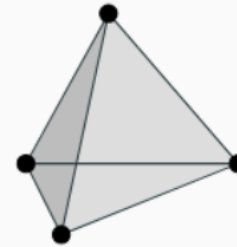
0-simplex



1-simplex



2-simplex



3-simplex

Definition 2.4

An **abstract simplicial complex** K consists of vertices, V_K , and a set S_K of nonempty subsets of V_K called simplices satisfying the following:

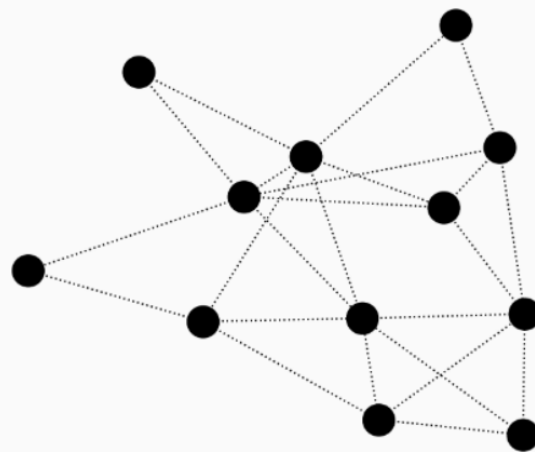
1. For any simplex $\sigma \subseteq V_K$, if $\tau \subseteq \sigma$, then $\tau \in S_K$, and
2. For each $v \in V_K$, $\{v\} \in S_K$.

Vietoris Rips Complex

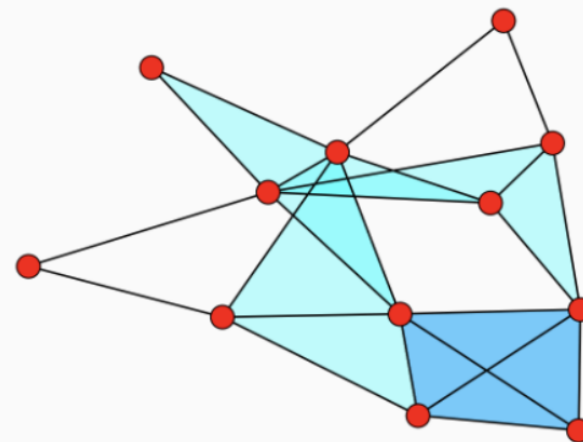
Definition 2.6

Let X be a metric space with distance function d . The **Vietoris–Rips complex** with parameter $\epsilon \geq 0$, denoted $\text{VR}((X, d); \epsilon)$ or simply $\text{VR}(X; \epsilon)$, is the simplicial complex defined as follows:

1. X is the vertex set of $\text{VR}((X, d); \epsilon)$, and
2. A finite subset $\sigma \subseteq X$ is the vertex set of a simplex if $\text{diam}(\sigma) \leq \epsilon$.



Underlying graph G



$\text{VR}((G, d_G); 1)$

Topological data analysis

- ▶ Using simplicial complexes to reveal topological structure of data
- ▶ Use of custom distance metrics allows for more flexibility and tailored approaches
- ▶ Still early on in this field so applications are limited but with growing access to data and complexity of data, these techniques could become more useful moving forward.



Thank you

SPECIAL THANK TO JOHN STERLING FOR MENTORING ME THROUGH THIS
PROCESS