1. Which of the following is a formula for a line which passes through the point $P = (1, 2, 3)$ and which is perpendicular to the vectors $\mathbf{v} = \langle 1, 0, 2\rangle$ and $\mathbf{w} = \langle 0, 3, -1\rangle$?

Proof. Since the line is perpendicular to both $\mathbf{v}$ and $\mathbf{w}$, a direction vector for the line is given by their cross product: $\mathbf{v} \times \mathbf{w} = \langle -6, 1, 3\rangle$.

Therefore, a vector equation for the line through $P = (1, 2, 3)$ is

$$\ell : \ \mathbf{r}(t) = \langle 1, 2, 3\rangle + t\langle -6, 1, 3\rangle,$$

where t is a real number ($t \in \mathbb{R}$). □

2. Find the area of the triangle with vertices $P = (2, 1, 1)$, $Q = (1, 2, 1)$, and $R = (1, 1, 2)$.

Proof. Recall that the area of a triangle determined by two side vectors $\mathbf{v}$ and $\mathbf{w}$ is

$$\frac{1}{2}|\mathbf{v} \times \mathbf{w}|.$$

Using P as the common base point, we have

$$\overrightarrow{PQ} = Q - P = \langle 1 - 2, 2 - 1, 1 - 1\rangle = \langle -1, 1, 0\rangle,$$

and

$$\overrightarrow{PR} = R - P = \langle 1 - 2, 1 - 1, 2 - 1\rangle = \langle -1, 0, 1\rangle.$$

Since

$$\left|\overrightarrow{PQ} \times \overrightarrow{PR}\right| = \sqrt{3},$$

the area of the triangle is $\frac{\sqrt{3}}{2}$.

Note A: Many of you mixed base points when choosing vectors, for example using $\overrightarrow{PQ}$ and $\overrightarrow{QR}$, or reversed the direction of one or more vectors. In this problem, these choices still produce the correct area because the magnitude of the relevant cross product remains unchanged.

However, if you want the two vectors to directly represent two sides of the original triangle when drawn tail-to-tail, they must have the same initial point. For example, $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ both begin at P. By contrast, $\overrightarrow{PQ}$ and $\overrightarrow{QR}$ do not share a common initial point, so placing them tail-to-tail generally produces a different triangle, even though the cross product still gives the same area.

Be careful with direction as well. Recall that

$$\overrightarrow{PQ} = Q - P,$$

where P and Q are interpreted as position vectors. □