

1. Find the equation of the plane that passes through the point $(1, -1, 2)$ and is perpendicular to both

$$2x + y - 2z = 1 \quad \text{and} \quad x + 3z = 10.$$

Proof. The normal vectors of the given planes are

$$\mathbf{n}_1 = \langle 2, 1, -2 \rangle \quad \text{and} \quad \mathbf{n}_2 = \langle 1, 0, 3 \rangle.$$

Since the desired plane is perpendicular to both planes, $\mathbf{n}_1$ and $\mathbf{n}_2$ are parallel to the desired plane.

Hence, $\mathbf{n}_1 \times \mathbf{n}_2$ is normal to the desired plane.

$$\mathbf{n} = \mathbf{n}_1 \times \mathbf{n}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1 & -2 \\ 1 & 0 & 3 \end{vmatrix} = \langle 3, -8, -1 \rangle.$$

Using the point $(1, -1, 2)$, the equation of the plane is

$$\langle 3, -8, -1 \rangle \cdot \langle x - 1, y + 1, z - 2 \rangle = 0.$$

Simplifying,

$$3x - 3 - 8y - 8 - z + 2 = 0,$$

so

$$\boxed{3x - 8y - z = 9}.$$

Therefore, the correct answer is E. □

2. Identify the surface

$$2x^2 + 3z^2 = 4x + 2y^2$$

through completing the square.

Proof. Rearranging,

$$2x^2 - 4x + 3z^2 - 2y^2 = 0.$$

Complete the square in x :

$$2(x^2 - 2x) + 3z^2 - 2y^2 = 0,$$

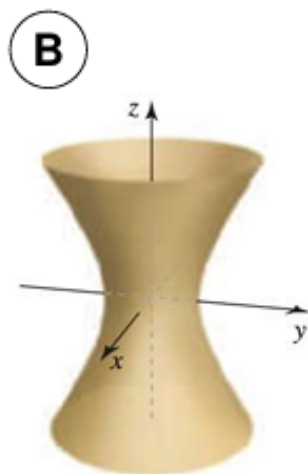
$$2((x - 1)^2 - 1) + 3z^2 - 2y^2 = 0,$$

$$2(x - 1)^2 + 3z^2 - 2y^2 = 2.$$

Dividing by 2 gives

$$(x - 1)^2 + \frac{3}{2}z^2 - y^2 = 1.$$

This equation has two positive squared terms and one negative squared term, so the surface is a hyperboloid of one sheet. Therefore, the correct answer is B .



□