1. Find the domain of the function

$$r(t) = \left\langle \sqrt{t^2 - 4t + 3},\, e^{3t},\, \ln\left(t^{1/3} - 1\right) \right\rangle.$$

Proof. For the square-root component, we require

$$t^2 - 4t + 3 \geq 0.$$

Factoring,

$$(t-1)(t-3) \geq 0,$$

so

$$t \leq 1 \quad \text{or} \quad t \geq 3.$$

For the logarithmic component, we require

$$t^{1/3} - 1 > 0.$$

Thus,

$$t^{1/3} > 1,$$

and hence,

$$t > 1.$$

The exponential component e^{3t} is defined for all real t.

Therefore the domain is

$$((-\infty, 1] \cup [3, \infty)) \cap (1, \infty) = [3, \infty).$$

Hence, $t \geq 3$.

□

2. A particle has acceleration $a(t) = \langle 0, 2t, \sqrt{2} \rangle$ with an initial velocity of $\langle 1, 0, 0 \rangle$ at $t = 0$. Find the distance traveled for $0 \leq t \leq 3$.

Proof. Integrating $a(t)$ yields

$$\mathbf{v}(t) = \left\langle C_1,\, t^2 + C_2,\, \sqrt{2}\,t + C_3 \right\rangle.$$

Using the initial condition,

$$\mathbf{v}(0) = \langle 1, 0, 0 \rangle,$$

we obtain

$$C_1 = 1, \qquad C_2 = 0, \qquad C_3 = 0.$$

Thus,

$$\mathbf{v}(t) = \langle 1, t^2, \sqrt{2}\,t \rangle.$$

The speed is

$$\begin{aligned}
|\mathbf{v}(t)| &= \sqrt{1^2 + (t^2)^2 + (\sqrt{2}\,t)^2} \\
&= \sqrt{1 + t^4 + 2t^2} \\
&= \sqrt{(t^2+1)^2} \\
&= t^2 + 1.
\end{aligned}$$

Therefore the distance traveled is

$$\begin{aligned}
D &= \int_0^3 |\mathbf{v}(t)|\,dt \\
&= \int_0^3 (t^2+1)\,dt \\
&= \left[\frac{t^3}{3} + t\right]_0^3 \\
&= 9 + 3 = 12
\end{aligned}$$

□