1. Compute the line integral $\int_C F \cdot dr$ where $F = \langle yz, xz, xy \rangle$ is conservative and the curve C is parameterized by $r(t) = \langle t^2, t, t^3 - 3t \rangle$, $1 \leq t \leq 2$.

Proof. Firstly, note that we could compute the integral by using the provided parameterization. However, I will use the Fundamental Theorem for Line Integrals we discussed in class. Since $F = \langle yz, xz, xy \rangle = \nabla \varphi = \langle \varphi_x, \varphi_y, \varphi_z \rangle$, then we can immediately note that $\varphi(x, y, z) = xyz$. From the given parameterization, our curve starts at $r(1) = (1, 1, -2)$ and ends at $r(2) = (4, 2, 2)$. Therefore, by the Fundamental Theorem for Line integrals

$$\int_C F \cdot dr = \varphi(4, 2, 2) - \varphi(1, 1, -2) = 16 - (-2) = 18,$$

so the correct option is B. □

2. According to Green's Theorem, which of the following line integrals is not equal to the area of the region enclosed by a simple curve C?

Proof. We are given the options

A) $\int_C xdy$

B) $\frac{1}{5} \int_C (4ydx - xdy)$

C) $\int_C -ydx$

D) $\frac{1}{3} \int_C (ydx + 4xdy)$

E) $\frac{1}{2} \int_C (-ydx + xdy)$.

Let D be the region enclosed by C. By Green's Theorem, we know that

$$\int_C f(x, y)dx + g(x, y)dy = \iint_D (g_x - f_y)dA.$$

For the integral to be equal to the area of D, the double integral resulting from Green's Theorem must have integrand 1. Thus, we need only check $g_x - f_y = 1$ in each case.

A) $\int_C xdy \implies g_x - f_y = 1 - 0 = 1$

B) $\frac{1}{5} \int_C (4ydx - xdy) \implies \frac{1}{5}(g_x - f_y) = \frac{1}{5}(-1 - 4) = -1$

C) $\int_C -ydx \implies g_x - f_y = 0 - (-1) = 1$

D) $\frac{1}{3} \int_C (ydx + 4xdy) \implies \frac{1}{3}(g_x - f_y) = \frac{1}{3}(4 - 1) = 1$

E) $\frac{1}{2} \int_C (-ydx + xdy) \implies \frac{1}{2}(g_x - f_y) = \frac{1}{2}(1 - (-1)) = 1$.

So, the correct option is B.

$\square$