1. Find a vector function that represents the curve of intersection of the cylinder $y^2 + z^2 = 1$ and the plane $x + y + 2z = 3$.

Proof. A convenient parametrization of the cylinder $y^2 + z^2 = 1$ is

$$y = \cos t, \qquad z = \sin t, \qquad 0 \le t < 2\pi.$$

Substitute into the plane $x + y + 2z = 3$:

$$x = 3 - y - 2z = 3 - \cos t - 2\sin t.$$

Therefore, a vector function for the curve of intersection is

$$\mathbf{r}(t) = \langle 3 - \cos t - 2\sin t,\ \cos t,\ \sin t\rangle, \qquad 0 \le t < 2\pi.$$

$\square$

2. Find the angle θ between the velocity and acceleration at $t = 1$ for the position vector $r(t) = \langle \cos t, \frac{t^2}{2}, -\sin t\rangle$.

Proof. We first compute the velocity, $r'(t)$, and acceleration $r''(t)$.

$$r'(t) = \langle -\sin t, t, -\cos t\rangle$$

and

$$r''(t) = \langle -\cos t, 1, \sin t\rangle.$$

Recall that the dot product yields $r'(t) \cdot r''(t) = |r'(t)||r''(t)| \cos\theta$, where θ is the angle between the two vectors $r'(t)$ and $r''(t)$. Hence, $\theta = \cos^{-1}\left(\frac{r'(t)\cdot r''(t)}{|r'(t)||r''(t)|}\right)$. The respective magnitudes are

$$|r'(t)| = \sqrt{\sin^2 t + t^2 + \cos^2 t} \overset{(i)}{=} \sqrt{t^2 + 1}$$

and

$$|r''(t)| = \sqrt{\cos^2 t + 1 + \sin^2 t} = \sqrt{2},$$

where (i) holds as $\sin^2 t + \cos^2 t = 1$. Since we are concerned with the angle at $t = 1$, we now evaluate each of the vector valued functions alongside their magnitudes at $t = 1$. It is easily seen that this produces $r'(1) = \langle -\sin 1, 1, -\cos 1\rangle$, $r''(1) = \langle -\cos 1, 1, \sin 1\rangle$, $|r'(1)| = \sqrt{2} = |r''(1)|$. Putting all of the pieces together,

$$\theta = \cos^{-1}\left(\frac{r'(1)\cdot r''(1)}{|r'(1)||r''(1)|}\right) = \cos^{-1}\left(\frac{\cos 1 \sin 1 + 1 - \cos 1 \sin 1}{\sqrt{2}\sqrt{2}}\right) = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}.$$

As an aside, you could also proceed using the cross product formula

$$|r'(t) \times r''(t)| = |r'(t)||r''(t)| \sin\theta.$$

This approach is far more tedious. $\square$