1. Consider the limits

$$A = \lim_{(x,y)\to(0,0)} \frac{3x-2y}{\sqrt{x^2+y^2}} \quad \text{and} \quad B = \lim_{(x,y)\to(0,0)} \frac{e^{x+y}}{1+e^{x-y}}.$$

Proof. To show nonexistence explicitly, take two paths:

$$\text{(i) } (x,y) = (x,0): \quad \frac{3x-2y}{\sqrt{x^2+y^2}} = \frac{3x}{|x|} \to 3 \quad (x \to 0^+).$$

$$\text{(ii) } (x,y) = (0,y): \quad \frac{3x-2y}{\sqrt{x^2+y^2}} = \frac{-2y}{|y|} \to -2 \quad (t \to 0^+).$$

Since the two path limits are different, the limit does not exist. Hence, A does not exist.

Next, the function

$$f(x,y) = \frac{e^{x+y}}{1+e^{x-y}}$$

is continuous everywhere because $e^{x-y} > 0$, so the denominator $1 + e^{x-y} \neq 0$. Therefore,

$$B = f(0,0) = \frac{e^{0+0}}{1+e^{0-0}} = \frac{1}{1+1} = \frac{1}{2}.$$

So,

$$B = \frac{1}{2}.$$

This gives us an answer of B. □

2. Let $r(t) = \left\langle t, \frac{t^2}{2}, \frac{t^3}{3} \right\rangle$, find $\kappa(1)$.

Proof. Curvature is given by

$$\kappa(t) = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}.$$

Differentiate:

$$\mathbf{r}'(t) = \left\langle 1, t, t^2 \right\rangle, \qquad \mathbf{r}''(t) = \langle 0, 1, 2t \rangle.$$

Compute the cross product:

$$\mathbf{r}'(t) \times \mathbf{r}''(t) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & t & t^2 \\ 0 & 1 & 2t \end{vmatrix} = \left\langle 2t^2 - t^2,\ -(2t),\ 1 \right\rangle = \left\langle t^2, -2t, 1 \right\rangle.$$

Hence, we have that

$$|\mathbf{r}'(t) \times \mathbf{r}''(t)| = \sqrt{t^4 + 4t^2 + 1}.$$

Also,

$$|\mathbf{r}'(t)| = \sqrt{1 + t^2 + t^4}.$$

Therefore,

$$\kappa(t) = \frac{\sqrt{t^4 + 4t^2 + 1}}{(1 + t^2 + t^4)^{3/2}}.$$

Evaluate at $t = 1$:

$$\kappa(1) = \frac{\sqrt{1 + 4 + 1}}{(1 + 1 + 1)^{3/2}} = \frac{\sqrt{6}}{3^{3/2}} = \frac{\sqrt{6}}{3\sqrt{3}} = \frac{\sqrt{2}}{3}.$$

So,

$$\kappa(1) = \frac{\sqrt{2}}{3},$$

which gives us an answer of D. $\square$