1. Evaluate the integral $\iint_D 2\pi\sin(x^2)dA$ where D is the region in the xy-plane bounded by the lines $y=0, y=x$, and $x=\sqrt{\pi}$.

Proof. After drawing out our lines, it is easy to see that our bounds are $0\le x\le\sqrt{\pi}$ and $0\le y\le x$. Hence, the iterated integral is $\int_0^{\sqrt{\pi}}\int_0^x 2\pi\sin(x^2)dydx$. Computing, we have that

$$\int_0^{\sqrt{\pi}}\int_0^x 2\pi\sin(x^2)dydx=\int_0^{\sqrt{\pi}}[2\pi\sin(x^2)y]_0^x dx=\int_0^{\sqrt{\pi}}2\pi\sin(x^2)xdx.$$

Letting $u=x^2$, $du=2x$, so via substitution the above integral becomes

$$\int_0^{\sqrt{\pi}}\int_0^x 2\pi\sin(x^2)dydx=\int_0^{\sqrt{\pi}}2\pi\sin(x^2)xdx=\pi\int_0^{\pi}\sin(u)du=\pi[-\cos(u)]_0^{\pi}=\pi(1-(-1))=2\pi.$$

Thus, the correct option is A. ☐

2. Evaluate the double integral $\iint_D 2e^{(x^2+y^2)}dA$ where D is the region bounded by the x-axis and the curve $y=\sqrt{1-x^2}$.

Proof. Since $y=\sqrt{1-x^2}$ is the curve outlining the top portion of the unit circle $y^2+x^2=1$, then translating into polar coordinates will simplify our problem. Recall that, via the Pythagorean Theorem, $r^2=x^2+y^2$. Sketching this region, it is easy to see that our bounds are $0\le r\le 1$ and $0\le\theta\le\pi$, so our integral becomes $\int_0^{\pi}\int_0^1 2re^{r^2}drd\theta$, where the extra factor of r is introduced as we switched to polar integration. Evaluating, we have that

$$\int_0^{\pi}\int_0^1 2re^{r^2}drd\theta=\int_0^{\pi}[e^{r^2}]_0^1 d\theta=\int_0^{\pi}(e-1)d\theta=[\theta(e-1)]_0^{\pi}=\pi(e-1).$$

Thus, the correct option is B. ☐