

1. The triple integral

$$\int_{-3}^3 \int_0^{\sqrt{9-x^2}} \int_0^{\sqrt{x^2+y^2}} 8(x^2 + y^2) dz dy dx$$

when converted to cylindrical coordinates becomes ...

Proof. Our bounds give us the following inequalities:

$$-3 \leq x \leq 3, \quad 0 \leq y \leq \sqrt{9-x^2}, \quad 0 \leq z \leq \sqrt{x^2+y^2}.$$

In cylindrical coordinates, $r^2 = x^2 + y^2$, so $0 \leq z \leq \sqrt{r^2} = r$. In the xy -plane, $-3 \leq x \leq 3$ and $0 \leq y \leq \sqrt{9-x^2}$ tell us that the projection of our region is bounded by the upper semi-circle of radius 3. Hence, $0 \leq r \leq 3$ and $0 \leq \theta \leq \pi$. So, our integral becomes

$$\int_0^\pi \int_0^3 \int_0^r 8(r^2)r dz dr d\theta = \int_0^\pi \int_0^3 \int_0^r 8r^3 dz dr d\theta,$$

where the additional factor of r in the integrand is from our coordinate change. Therefore, the correct option is B . □

2. Which of the following integrals represents the volume of the solid in the first octant that is bounded on the side by the surface $x^2 + y^2 = 4$ and on top by the surface $x^2 + y^2 + z = 4$?

Proof. Since our solid is restricted to the first octant, then $x \geq 0, y \geq 0$, and $z \geq 0$. From our top bound, we have that $z = 4 - x^2 - y^2$, so $0 \leq z \leq 4 - x^2 - y^2$. In the xy -plane, our solid is bound by $x^2 + y^2 = 4$, i.e., the circle of radius 2. Since we are restricted to the first octant, then we have a semi-circle with radius 2, so $0 \leq y \leq \sqrt{4-x^2}$ and $0 \leq x \leq 2$ sufficiently describes the region. Thus, the integral is

$$\int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{4-x^2-y^2} dz dy dx,$$

where we integrate over 1 as we want to recover the volume of the solid. Thus, the correct answer is C . □