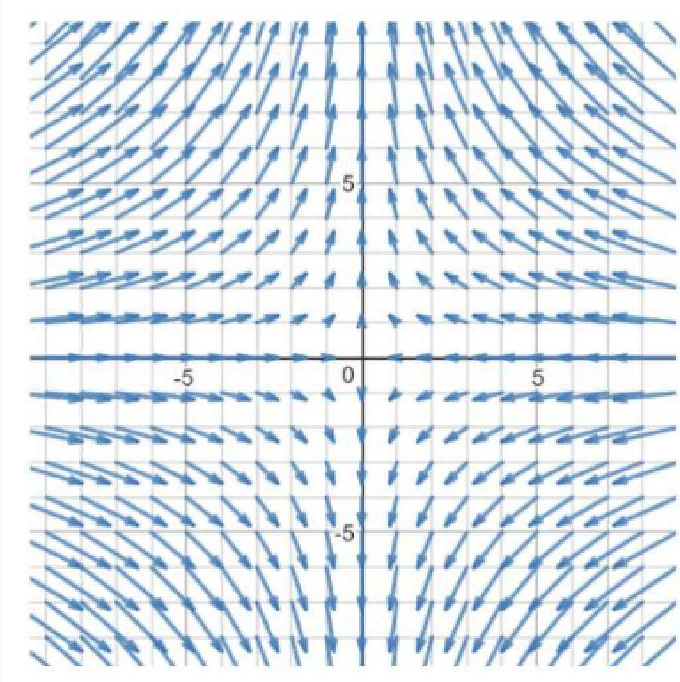
1. The graph below most represents the gradient field of which function.



Proof. Our options (alongside their gradients) are

a) $f(x,y) = y^2 + x^2 + 10 \implies \nabla f(x,y) = \langle 2x, 2y \rangle$

b) $f(x,y) = ye^x \implies \nabla f(x,y) = \langle e^x y, e^x \rangle$

c) $f(x,y) = \frac{y}{x} \implies \nabla f(x,y) = \langle -y/x^2, 1/x \rangle$

d) $f(x,y) = y^2 - x^2 - 10 \implies \nabla f(x,y) = \langle -2x, 2y \rangle$

e) $f(x,y) = xe^y \implies \nabla f(x,y) = \langle e^y, xe^y \rangle$.

Looking at the point $(1,0)$, **a)** outputs $\langle 2, 0 \rangle$, which doesn't match our graph. Similarly, **b)** outputs $\langle 0, e \rangle$, **c)** outputs $\langle 0, 1 \rangle$, and **d)** outputs $\langle 1, 1 \rangle$, and each of these outputs do not match out graph. Thus, the only option is **d**. □

2. Let C be the curve $r(t) = \langle \cos t, \sin t, t \rangle$, $t \in [0, \pi/2]$ and $f(x,y,z) = xy$, then what is $\int_C f(x,y,z)ds$?

Proof. Since $ds = |r'(t)|dt$, then in terms of our parameterization

$$\int_C f(x,y,z)ds = \int_0^{\pi/2} f(r(t))|r'(t)|dt.$$

Since $r'(t) = \langle -\sin t, \cos t, 1 \rangle$, then $|r'(t)| = \sqrt{\sin^2 t + \cos^2 t + 1} = \sqrt{2}$. Moreover, by definition, $f(r(t)) = \cos t \sin t$. Thus, our integral becomes

$$\int_0^{\pi/2} f(r(t))|r'(t)|dt = \int_0^{\pi/2} \sin t \cos t \sqrt{2} dt.$$

Letting $u = \sin t$, then $du = \cos t dt$, so we have that

$$\int_0^{\pi/2} \sin t \cos t \sqrt{2} dt = \sqrt{2} \int_0^1 u du = \sqrt{2} \left[\frac{u^2}{2} \right]_0^1 = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}.$$

Therefore, the correct option is D. □